Testing Bell inequalities with circuit QEDs by joint spectral measurements

Hao Yuan,[1,2] L. F. Wei,[1,3,*] J. S. Huang,[1] X. H. Wang,[1] and Vlatko Vedral[2,4,5,†]

[1] *Quantum Optoelectronics Laboratory, School of Physics and Technology, Southwest Jiaotong University, Chengdu 610031, China*
[2] *Clarendon Laboratory, University of Oxford, Parks Road, Oxford OX1 3PU, United Kingdom*
[3] *State Key Laboratory of Optoelectronic Materials and Technologies, School of Physics and Engineering, Sun Yat-Sen University, Guangzhou 510275, China*
[4] *Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543, Singapore*
[5] *Department of Physics, National University of Singapore, 2 Science Drive 3, Singapore 117542, Singapore*

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We propose a feasible approach to test Bell's inequality with the experimentally-demonstrated circuit QED system, consisting of two well-separated superconducting charge qubits (SCQs) dispersively coupled to a common one-dimensional transmission line resonator (TLR). Our proposal is based on the joint spectral measurements of the two SCQs, i.e., their quantum states in the computational basis $\{|kl\rangle,\ k,l=0,1\}$ can be measured by detecting the transmission spectra of the driven TLR: each peak marks one of the computational basis and its relative height corresponds to the probability superposed. With these joint spectral measurements, the generated Bell states of the two SCQs can be robustly confirmed without the standard tomographic technique. Furthermore, the statistical nonlocal-correlations between these two distant qubits can be directly read out by the joint spectral measurements, and consequently the Bell's inequality can be tested by sequentially measuring the relevant correlations related to the suitably-selected sets of the classical local variables $\{\theta_j,\theta_j',j=1,2\}$. The experimental challenges of our proposal are also analyzed.

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I. INTRODUCTION

Historically, the well-known Einstein, Podolsky, and Rosen (EPR) paradox [1] concerning the completeness of quantum mechanics was proposed based on a gedanken experiment in 1935. EPR claimed that quantum mechanics is incomplete and so-called local hidden variables (LHV) should exist. Subsequently, it has provoked much debate on the completeness of quantum mechanics and the existence of LHV theories. In 1964, Bell actually quantified the EPR argument and derived a strict inequality [2] with respect to the correlation strengths possible to achieve by all the LHV models. If this inequality is violated, then there are no LHV and the quantum mechanical prediction of the existence of quantum nonlocal correlation (i.e., entanglement) is sustained. However, Bell's seminal inequality did not allow for the practically-existing imperfections and thus was not accessible to the experimental tests. Soon later, Clauser, Horne, Shimony and Holt (CHSH) addressed this issue and derived an experimentally testable inequality [3]. During the past decades, a number of experimental tests of CHSH-type Bell's inequality have been reported using, e.g., entangled photon pairs [4], trapped ions [5], an atom and a photon [6], two superconducting phase qubits [7], and even two entangled degrees of freedom (comprising spatial and spin components) of single neutrons [8], and so on. These experimental evidences strongly convince that Bell's inequality could be violated and thus agree well with quantum mechanical predictions, ruling out the LHV theories.

Recently, a novel superconducting electrical circuit architecture (called circuit QED) analogous to cavity QED has been first suggested by Blais et al. [9], and then realized in the experiment by Wallraff et al.[10]. In circuit QED, a superconducting qubit is strongly coupled to one-dimensional transmission line resonator (TLR) which acts as quantized cavity. Now much attention has been focused on this field because it opens the possibility of studying quantum optics phenomena in solid-state system and quantum information processing. Experimentally, remarkable advances have been achieved such as: observation of the vacuum Rabi splittings [10] and AC Stack shifts [11], observing Berry's phase [12], measuring the Lamb shifts [13], generations of microwave single photons [14] and Fock states [15], coupling two qubits via cavity as quantum bus [7, 16], two-qubit Grover and Deutsch-Jozsa algorithms [17], etc. In this system the qubit-state readouts were achieved by detecting the state-dependent shifts of the frequency of the dispersively-coupled resonator [17, 19–22]. Indeed, in the dispersive regime [18], the transition frequencies of the superconducting qubits are far detuned from the frequency of the coupled resonator. As a consequence, a sufficiently large state-dependent frequency shifts of the resonator can be induced [9, 10], which could be tested by measuring the transmission spectra of the driven resonator.

Very recently, an efficient approach to implement the joint spectral measurements of the multi-qubits has been proposed by detecting the transmission spectra of the common resonator [23]. By considering all the quantum correlation of qubits and resonator (i.e., beyond the usual coarse-grained/mean-field approximation), it is shown that each peak in the measured transmission spectra of the driven resonator marks one of the logic states and the relative height of such a peak is related to its corresponding superposed probability. With these joint spectral measurements, Huang et al., [24] pre-

* Electronic address: weilianfu@gmail.com
† Electronic address: vlatko.vedral@qubit.org

sented a high efficiency tomographic reconstructions of more than one-qubit quantum states. As another application, in this paper we discuss how the proposed joint spectral measurements can be further utilized to test Bell's inequality with an experimentally-demonstrated two-qubit circuit QED system. Compared with the previous schemes for testing Bell inequalities, the advantage of the present proposal is that, the desirable coincided measurements on the two qubits can be directly implemented by the suitably-designed joint spectral measurements. This is because the states of the two qubits jointly determine the frequency shifts and consequently the spectral distributions of the driven resonator. Therefore, the nonlocal correlations between the well-separated qubits can be directly read out by analyzing the measured transmission spectra of the driven resonator. We note in passing that none of the circuit QED proposals so far can be used to close the locality loophole.

The paper is organized as follows: In section II, we propose a method to deterministically generate an ideal Bell state with the experimentally-demonstrated circuit QED system, via an *i*SWAP gate assisted by several single-qubit gates [25]. Further, a simple quantum interference method, instead of the usual quantum-state tomographic reconstruction, is proposed to confirm such a preparation by coherent quantum operations and projective measurements on two qubits in the computational basis. In section III, we discuss how to utilize the joint spectral measurements to implement this confirmation. Importantly, by applying two individual Hadamard-like operations to encode the local variables $\{\theta_1, \theta_2\}$ into the generated Bell state, we show that the statistical correlations between the distant qubits can be read out by these spectral measurements for various typical sets of classical local variables $\{\theta_j, \theta'_j, j=1,2\}$. With these measured correlations the CHSH-type Bell's inequality can be tested. Discussion and conclusion are presented in section IV.

II. GENERATION AND CONFIRMATION OF BELL STATES

In this section, we will show how to deterministically prepare one of the ideal Bell states

$$|\Phi_\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), |\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle), \quad (1)$$

with the experimentally-demonstrated circuit QED system. In fact, the Bell state in this system have been prepared by several methods, including measuring-based synthesis [26] and gate sequence (see, eg. Refs. [7, 16, 19, 20, 25]). Here, we will present an alternative gate sequence method to implement the desirable preparation based on an *i*SWAP gate (the previous methods use the controlled phase gate).

We consider the circuit QED architecture sketched in Fig.1, in which two superconducting charge qubits (SCQs) are coupled to one-dimensional TLR. To obtain the maximal coupling, two SCQs are placed close to the ends of the resonator (voltage antimodes of the resonator). When two SCQs work at their optimal point and under the rotating-wave approximation

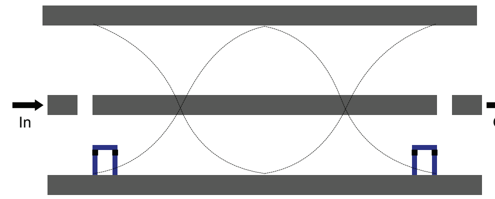

FIG. 1: (Color online) Circuit QED architecture with two superconducting charge qubits (SCQs) dispersively coupling to one-dimensional transmission line resonator (TLR). The joint spectral measurements of the two SCQs can be realized by detecting the state-dependent frequency shifts in the transmission spectra of the driven resonator.

(RWA), the system of two SCQs plus resonator is described by the usual Dicke Hamiltonian ($\hbar = 1$) [25]

$$H = \omega_r a^\dagger a + \sum_{j=1,2}[\frac{\omega_j}{2}\sigma_{zj} + g_j(a^\dagger\sigma_{-j} + a\sigma_{+j})]. \quad (2)$$

Here, ω_r is the single-mode frequency of the TLR, $a^\dagger(a)$ its creation (annihilation) operator; ω_j the transition frequency of jth SCQ that can be adjusted by external bias flux and g_j the coupling strength of the jth SCQ to the resonator. Finally, $\sigma_{+j} = |1\rangle_j\langle 0|$, $\sigma_{-j} = |0\rangle_j\langle 1|$ and $\sigma_{zj} = |0\rangle_j\langle 0| - |1\rangle_j\langle 1|$. Two SCQs in this circuit can be controlled and measured by driving the resonator. The Hamiltonian to describe such a drive is given by

$$H_d = \epsilon(a^\dagger e^{-i\omega_d t} + a e^{i\omega_d t}), \quad (3)$$

where ϵ is time-independent real amplitude and ω_d the frequency of the externally applied drive.

From the easily-prepared initial state $|00\rangle_{12}$, an ideal Bell state can be generated deterministically by the following three steps, the corresponding gate sequence is shown in Fig. 2.

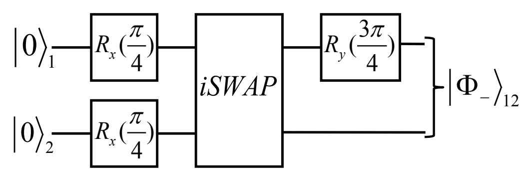

FIG. 2: Gate sequence to generate the Bell state $|\Phi_-\rangle_{12}$ from the initial state $|00\rangle_{12}$.

Step 1: Perform the unitary operation $R_x(\pi/4) = \exp(i\sigma_x\pi/4)$ on each SCQ to prepare the superposition of two basis states of each SCQ. That is,

$$|00\rangle_{12} \overset{R_{x1}(\frac{\pi}{4})R_{x2}(\frac{\pi}{4})}{\rightarrow} |\psi\rangle_{12} = \frac{1}{2}(|0\rangle + i|1\rangle)_1(|0\rangle + i|1\rangle)_2. (4)$$

The unitary operations $R_{x1}(\pi/4)$ and $R_{x2}(\pi/4)$ can be achieved by externally applied microwave drives with the duration of $t_1 = -\pi\Delta_r/4\epsilon g_1$ and $t_2 = -\pi\Delta_r/4\epsilon g_2$, respectively [25]. Here, $\Delta_r = \omega_d - \omega_r$ is the detuning between the drive frequency and the resonator one.

Step 2: In the dispersive regime, i.e.,$|g_1/\Delta_1| \ll 1$ and $|g_2/\Delta_2| \ll 1$ (with $\Delta_j = \omega_j - \omega_r$ being the detuning between the jth SCQ and the TLR), and $\Delta_1 = \Delta_2 = \Delta$ and $g_1 = g_2 = g$ (achieved by adjusting the external bias flux), the efficient Hamiltonian of this system reads

$$\begin{aligned} H_{eff} &= -\frac{g^2}{2\Delta}[4(a^\dagger a + \frac{1}{2})(\sigma_{z1} + \sigma_{z2}) \\ &\quad -(\sigma_{x1}\sigma_{x2} + \sigma_{y1}\sigma_{y2})]. \end{aligned} \quad (5)$$

This Hamiltonian generates a two-qubit gate (with the evolution duration $t_s = 3\pi\Delta/2g^2$)

$$U(t_s) = \exp(-iH_{eff}t_s) = \begin{pmatrix} e^{i\delta} & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & e^{-i\delta} \end{pmatrix}, \quad (6)$$

where $\delta = 6\pi(n + 1/2)$ with $n = \langle a^\dagger a\rangle$ being the mean photon number in the resonator. If n is known in advance, the global phase factor δ can be eliminated by performing single-qubit phase rotations: $|0\rangle \to e^{-i\delta/2}|0\rangle$ and $|1\rangle \to e^{i\delta/2}|1\rangle$, on each qubit. Then, the above two-qubit gate reduces to the desirable iSWAP gate

$$U_{iSWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (7)$$

Note that no external microwave drive is applied in this step. With such an *i*SWAP gate, the entanglement between two SCQs is generated, i.e.,

$$|\psi\rangle_{12} \overset{U_{iSWAP}}{\to} |\psi'\rangle_{12} = \frac{1}{2}(|00\rangle - |01\rangle - |10\rangle - |11\rangle)_{12}. \quad (8)$$

Step 3: Apply the unitary operation $R_y(3\pi/4)$ to the first SCQ for generating a desirable Bell state $|\Phi_-\rangle_{12}$ from the state $|\psi'\rangle_{12}$ prepared above, i.e.,

$$|\psi'\rangle_{12} \overset{R_{y1}(\frac{3\pi}{4})}{\to} |\Phi_-\rangle_{12} = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)_{12}. \quad (9)$$

Here, the unitary operation $R_y(3\pi/4)$ is implemented by $R_y(3\pi/4) = R_x(\pi/4)R_z(3\pi/4)R_x(3\pi/4)$. $R_z(3\pi/4)$ and $R_x(3\pi/4)$ can be realized by applying microwave drives with the duration $t_3 = 3\pi\Delta_a/[2(\Delta_a + g_1^2/\Delta_1)\Delta_a + (2\epsilon g_1/\Delta_r)^2]$ ($\Delta_a = \omega_1 - \omega_d$) and $t_4 = -3\pi\Delta_r/4\epsilon g_1$, respectively.

Similarly, other Bell states in Eq.(1) can be also generated with the above method.

Customarily, the quantum-state preparation is experimentally confirmed by quantum-state tomography (see, e.g., [7, 17, 19, 20, 22]), i.e., reconstructing its density matrix via a series of measurements on many copies of the prepared state. However, based on the same logic proposed in Ref. [27] the Bell state generated above can be simply confirmed by the following two steps. First, we perform a projective measurement on the prepared state in the computational basis. The outputs will be either the logic state $|00\rangle$ or the $|11\rangle$ state with the same probability (i.e., $P_{00} = P_{11} = 1/2$). Note that this is not the sufficient condition for the confirmation of the Bell state $|\Phi_-\rangle_{12}$, since a statistical mixture of the logic states $|00\rangle$ and $|11\rangle$ with the same probability $1/2$ may also result in the same outputs. Therefore, besides the above projective measurement performed directly, another projective measurement in the computational basis is also required after the unitary operation $R_y(\pi/4) = R_z(\pi/4)R_x(3\pi/4)R_z(3\pi/4)$ on each SCQ. These operations evolve the prepared Bell state to another Bell state, i.e.,

$$|\Phi_-\rangle_{12} \overset{R_{y1}(\frac{\pi}{4})R_{y2}(\frac{\pi}{4})}{\to} |\Psi_+\rangle_{12} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)_{12}. \quad (10)$$

This measurement implies that, if the prepared state is the desirable Bell state $|\Phi_-\rangle_{12}$, then the measured state should be $|\Psi_+\rangle_{12}$ and thus the output is either $|01\rangle$ or $|10\rangle$ (with the same probability $1/2$). This is because the mixture of the states $|00\rangle$ and $|11\rangle$ would not vanish by the quantum interference after the designed quantum operations.

III. JOINT SPECTRAL MEASUREMENTS OF TWO QUBITS FOR TESTING BELL'S INEQUALITY

A. Joint spectral measurements of the qubits for confirming the preparation of Bell states

Customarily, completely characterizing an unknown quantum state requires to tomographically reconstruct its density matrix, by a series of quantum measurements performed on its many copies (see, e.g., [7, 17, 19, 20, 22]). Here, we will show that the Bell state generated above, e.g., $|\Phi_-\rangle_{12}$, can be confirmed by using the joint spectral measurements of the two qubits in a significantly simpler fashion. In the dispersive regime of two-qubit circuit QED system, the transition frequency of each SCQ is far detuned from the coupled resonator. And in a framework rotating at the drive frequency ω_d, the efficient Hamiltonian of the whole system is

$$\begin{aligned} \mathcal{H} &= (-\Delta_r + \Gamma_1\sigma_{z1} + \Gamma_2\sigma_{z2})a^\dagger a + \frac{\tilde{\omega}_1}{2}\sigma_{z1} + \frac{\tilde{\omega}_2}{2}\sigma_{z2} \\ &\quad +\epsilon(a^\dagger + a), \end{aligned} \quad (11)$$

where $\Gamma_j = g_j^2/\Delta_j$ and $\tilde{\omega}_j = \omega_j + \Gamma_j$, $j = 1, 2$. From the first term in Eq. (11), it can be directly seen that the resonator is pulled by different logic states of two qubits. The frequency shifts of the resonator by $-\Gamma_1 - \Gamma_2$, $-\Gamma_1 + \Gamma_2$, $\Gamma_1 - \Gamma_2$, and $\Gamma_1 + \Gamma_2$ correspond to the logic states of two qubits $|11\rangle$, $|10\rangle$, $|01\rangle$, and $|00\rangle$, respectively. Thus, the frequency shifts of the resonator can be used to mark the logic states of two qubits.

Further, by considering all the statistical quantum correlations between SCQs and resonator (i.e., beyond the usual coarse-grained/mean-field approximation), the steady-state transmission spectra of the driven resonator $S_{ss} = \langle a^\dagger a\rangle_{ss}/2\epsilon$ can be analytically derived as [23, 24]

$$S_{ss} = -\frac{2(AC + BD)}{\kappa(A^2 + B^2)}, \quad (12)$$

with $A = (\Gamma_1^2 - \Gamma_2^2) + 2(\kappa^2/4 - \Delta_r^2)(\Gamma_1^2 + \Gamma_2^2) + (\kappa^2/4 - \Delta_r^2)^2 - \kappa^2\Delta_r^2$, $B = -2\kappa\Delta_r(\Gamma_1^2 + \Gamma_2^2 + \kappa^2/4 - \Delta_r^2)$, $C = \kappa\langle\sigma_{z1}(0)\sigma_{z2}(0)\rangle_2\Gamma_1\Gamma_2 - \kappa\Delta_r(\langle\sigma_{z1}(0)\rangle_2\Gamma_1 + \langle\sigma_{z2}(0)\rangle_2\Gamma_2) + \kappa/2(3\Delta_r^2 - \kappa^2/4 - \Gamma_1^2 - \Gamma_2^2)$, and $D = -2\langle\sigma_{z1}(0)\sigma_{z2}(0)\rangle_2\Delta_r\Gamma_1\Gamma_2 - \sum_{j=1}^2\langle\sigma_{zj}(0)\rangle_j\Gamma_j(\Gamma_j^2 - \Gamma_{j'}^2 + \kappa^2/4 - \Delta_r^2) + \Delta_r(\Gamma_1^2 + \Gamma_2^2 + 3\kappa^2/4 - \Delta_r^2)$, $j \neq j' = 1, 2$, $\Gamma_j = g_j^2/\Delta_j$. Here, κ denotes photon leakage rate of the resonator. Experimentally, the qubit decay rate (e.g., $\gamma = 2\pi \times 0.25$MHz [28]) is negligible, since it is significantly smaller than the photon leakage rate of the resonator (e.g., $\kappa = 2\pi \times 1.69$MHz [22]). It has been numerically proven that, each peak in the above steady-state spectra marks one of the logic states and the relative height of such a peak corresponds to the superposed probability in the detected two-qubit state.

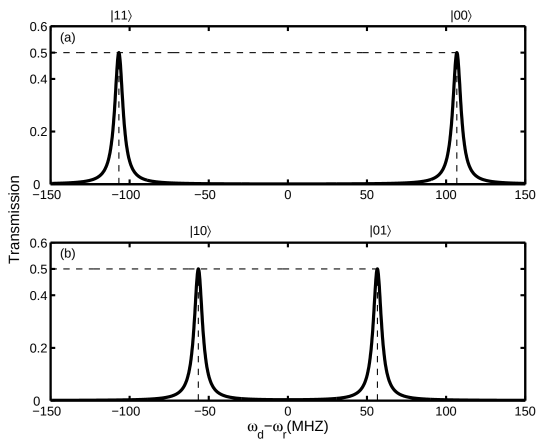

FIG. 3: Transmission spectra of the driven resonator versus the detuning $\Delta_r = \omega_d - \omega_r$ for Bell states $|\Phi_-\rangle_{12}$ (a) and $|\Psi_+\rangle_{12}$ (b). The parameters are chosen as $(\Gamma_1, \Gamma_2, \kappa) = 2\pi \times (13, 4, 1)$MHz [20].

Therefore, if the two SCQs are prepared at the state $|\Phi_-\rangle_{12}$, then two peaks marking respectively $|00\rangle$ and $|11\rangle$ with the same height would be detected in the measured spectra. Furthermore, if the joint spectral measurements are performed after the evolution (10), then other two peaks marking respectively $|10\rangle$ and $|01\rangle$ should be detected in the spectra. This is the direct evidence showing that the state generated above is just the desirable Bell state, i.e., the coherent superposition of the states $|00\rangle$ and $|11\rangle$, rather than their mixture. This argument could be verified by the numerical results shown in Fig. 3. For the typical experimental parameters $(\Gamma_1, \Gamma_2, \kappa) = 2\pi \times (13, 4, 1)$MHz [20], the Fig. 3(a) shows the steady-state spectral distributions (versus the detuning $\Delta_r = \omega_d - \omega_r$) for the Bell state $|\Phi_-\rangle_{12}$, and Fig. 3(b) for another Bell state $|\Psi_+\rangle_{12}$.

B. Testing Bell's inequality via joint spectral measurements

Being able to measure Bell states constitutes a good entanglement witness, but we can in fact do better than that. With the generated Bell state $|\Phi_-\rangle_{12}$, we now show how to test Bell's inequality with circuit QED via joint spectral measurements of the distant SCQs.

First, with the single-qubit gates [25], a Hadamard-like operation [29] can be constructed as

$$\begin{aligned} R_j(\theta_j) &= R_z(\theta_j/2)R_x(\pi/4)R_z(-\theta_j/2) \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & ie^{i\theta_j} \\ ie^{-i\theta_j} & 1 \end{pmatrix}, \end{aligned} \tag{13}$$

which plays an important role for testing Bell's inequality.

Then, the classical variables $\{\theta_1, \theta_2\}$ can be encoded into the generated Bell state $|\Phi_-\rangle_{12}$ with Hadamard-like operations $R_j(\theta_j)$. Thus, the state $|\Phi_-\rangle_{12}$ is changed into

$$\begin{aligned} |\Phi'_-\rangle_{12} &= R_1(\theta_1)R_2(\theta_2)|\Phi_-\rangle_{12} \\ &= \frac{1}{2\sqrt{2}}[(1 + e^{-i(\theta_1+\theta_2)})|00\rangle + i(e^{i\theta_2} - e^{-i\theta_1})|01\rangle \\ &+ i(e^{i\theta_1} - e^{-i\theta_2})|10\rangle - (1 + e^{i(\theta_1+\theta_2)})|11\rangle]_{12}. \end{aligned} \tag{14}$$

Thirdly, the correlation between two classical variables $\{\theta_1, \theta_2\}$ is measured by the difference in the probabilities of two qubits found in the same and different logic states. This means that the correlation function $E(\theta_1, \theta_2)$ can be obtained as

$$\begin{aligned} E(\theta_1, \theta_2) &= P_{same}(\theta_1, \theta_2) - P_{diff}(\theta_1, \theta_2) \\ &= P_{00}(\theta_1, \theta_2) + P_{11}(\theta_1, \theta_2) - P_{01}(\theta_1, \theta_2) \\ &\quad - P_{10}(\theta_1, \theta_2), \end{aligned} \tag{15}$$

where $P_{00}(\theta_1, \theta_2)$ ($P_{01}(\theta_1, \theta_2)$, $P_{10}(\theta_1, \theta_2)$, $P_{11}(\theta_1, \theta_2)$) denotes the probability of two qubits found in the logic state $|00\rangle_{12}$ ($|01\rangle_{12}$, $|10\rangle_{12}$, $|11\rangle_{12}$).

Theoretically, the correlation of two classical variables $\{\theta_1, \theta_2\}$ can be described by the operator $T = |11\rangle\langle 11| + |00\rangle\langle 00| - |10\rangle\langle 01| - |01\rangle\langle 01| = \sigma_{z1} \otimes \sigma_{z2}$ and thus the correlation function is calculated as

$$E(\theta_1, \theta_2) =_{12} \langle\Phi'_-|T|\Phi'_-\rangle_{12} = \cos(\theta_1 + \theta_2). \tag{16}$$

Thus, for the set of classical local variables $\{\theta_1, \theta_2, \theta'_1, \theta'_2\}$, the so-called CHSH function [3] reads

$$\begin{aligned} f(|\Phi'_-\rangle_{12}) &= |E(\theta_1, \theta_2) + E(\theta'_1, \theta_2) + E(\theta_1, \theta'_2) \\ &\quad - E(\theta'_1, \theta'_2)| \\ &= |\cos(\theta_1 + \theta_2) + \cos(\theta'_1 + \theta_2) \\ &\quad + \cos(\theta_1 + \theta'_2) - \cos(\theta'_1 + \theta'_2)|. \end{aligned} \tag{17}$$

Typically, for one set of classical local variables $\{\theta_1, \theta_2, \theta'_1, \theta'_2\} = \{\pi/4, 3\pi/4, \pi/2, \pi\}$, the CHSH function [3] is calculated as

$$f(|\Phi'_-\rangle_{12}) = \sqrt{2} + 1 > 2. \tag{18}$$

Obviously, the CHSH-type Bell's inequality [3] $f \leq 2$ is violated. While, for another set of classical local variables $\{\theta_1, \theta_2, \theta'_1, \theta'_2\} = \{\pi/4, 0, 7\pi/4, 3\pi/2\}$, the CHSH function [3] is

$$f(|\Phi'_-\rangle_{12}) = 2\sqrt{2} > 2. \tag{19}$$

Therefore, in this case the CHSH-type Bell's inequality [3] $f \leq 2$ is violated maximally.

The above theoretical predictions can be numerically tested by the experimental joint spectral measurements. Indeed, the correlation function in Eq. (15) can be directly calculated from the measured transmission spectra of the driven resonator. For example, for the set of classical local variables

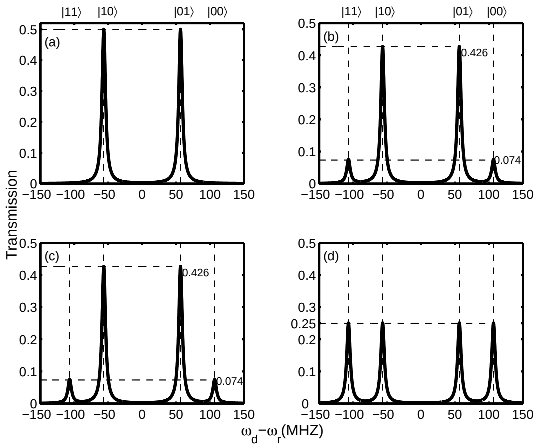

FIG. 4: Transmission spectra of the driven resonator versus the detuning $\Delta_r = \omega_d - \omega_r$ for the state $|\Phi'_{-}\rangle_{12}$ with one set of classical local variables $\{\theta_1, \theta_2, \theta_1', \theta_2'\} = \{\pi/4, 3\pi/4, \pi/2, \pi\}$. (a)-(d): the correlation functions can be directly calculated, $(E(\theta_1, \theta_2)$, $E(\theta_1', \theta_2)$, $E(\theta_1, \theta_2')$, $E(\theta_1', \theta_2')) = (-1, -0.704, -0.704, 0)$, according to Eq. (15). The parameters are are the same as that in Fig. 3.

$\{\theta_1, \theta_2, \theta_1', \theta_2'\} = \{\pi/4, 3\pi/4, \pi/2, \pi\}$, Fig. 4 shows the relevant transmission spectra (corresponding to the state $|\Phi'_{-}\rangle_{12}$) versus the detuning of $\Delta_r = \omega_d - \omega_r$. Here, the parameters are chosen as $(\Gamma_1, \Gamma_2, \kappa) = 2\pi \times (13, 4, 1)$MHz [20]. With the Fig. 4(a)-(d), the correlation functions between two classical local variables can be directly calculated $(E(\theta_1, \theta_2)$, $E(\theta_1', \theta_2)$, $E(\theta_1, \theta_2')$, $E(\theta_1', \theta_2')) = (-1, -0.704, -0.704, 0)$. Thus the CHSH function [3] is calculated as

$$f(|\Phi'_{-}\rangle_{12}) = 2.408 \approx \sqrt{2} + 1 > 2. \tag{20}$$

This indicates that the CHSH-type Bell's inequality [3] $f \leq 2$ is violated. Similarly, for another set of classical local variables $\{\theta_1, \theta_2, \theta_1', \theta_2'\} = \{\pi/4, 0, 7\pi/4, 3\pi/2\}$, with the same experimental parameters, we plot the transmission spectra of the driven resonator versus the detuning $\Delta_r = \omega_d - \omega_r$ for the state $|\Phi'_{-}\rangle_{12}$ in Fig. 5. Again, from the Fig. 5(a)-(d) the correlation functions are directly calculated as $(E(\theta_1, \theta_2)$, $E(\theta_1', \theta_2)$, $E(\theta_1, \theta_2')$, $E(\theta_1', \theta_2')) = (0.704, 0.704, 0.704, -0.704)$. As a consequence, we easily obtain the CHSH function [3]:

$$f(|\Phi'_{-}\rangle_{12}) = 2.816 \approx 2\sqrt{2} > 2. \tag{21}$$

Obviously, the CHSH-type Bell's inequality [3] $f \leq 2$ is significantly violated.

Finally, we emphasize that compared with the previous schemes for testing Bell's inequality (see, e.g., [7, 20]) with superconducting qubits, the present proposal seems much simpler and easier to be experimentally realized. We discuss its advantages next.

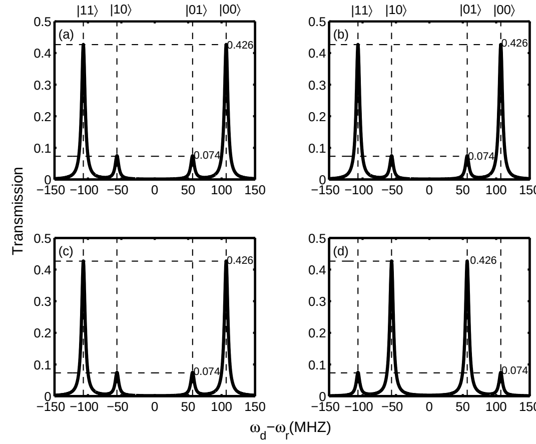

FIG. 5: Plot of the transmission spectra of the driven resonator versus the detuning $\Delta_r = \omega_d - \omega_r$ for the state $|\Phi'_{-}\rangle_{12}$ with another set of classical local variables $\{\theta_1, \theta_2, \theta_1', \theta_2'\} = \{\pi/4, 0, 7\pi/4, 3\pi/2\}$. (a)-(d): the correlation functions can be directly calculated as $(E(\theta_1, \theta_2)$, $E(\theta_1', \theta_2)$, $E(\theta_1, \theta_2')$, $E(\theta_1', \theta_2')) = (0.704, 0.704, 0.704, -0.704)$. The parameters are the same as that in Fig. 3.

IV. DISCUSSION AND CONCLUSION

We now briefly address the experimental feasibility of our scheme. With the typical experimental parameters $(\omega_r, \omega_1, \omega_2, g_{1(2)})$=$2\pi\times$(6.442, 4.5, 4.85, 0.133)GHz [19] and the amplitude of the drive chosen as ϵ=$2\pi\times$1.2GHz, we can approximately estimate the time needed in our scheme. The required times for realizing the unitary operatons $R_x(\pi/4)$, $R_x(3\pi/4)$ (setting $\omega_d = 2\pi\times$4.491GHz to satisfy $\Delta_a + \Gamma_j = 0$), $R_z(3\pi/4)$ (setting $\omega_d = 2\pi\times$4GHz), an iSWAP gate (setting $\Delta = 2\pi \times 1.18$GHz) are $t_{1(2)}$=1.5ns, t_4=4.5ns, t_3=1.5ns, and t_s=50ns, respectively. Experimentally, the time interval for performing a joint spectral measurement introduced above is about 40ns [22]. Thus, the times for generating a Bell state and confirming its existence are about 60.5ns and 93ns, respectively. Also, the time interval for testing Bell's inequality can be estimated as ~ 160ns. Finally, the total time in our scheme to perform a test experiment is about 313.5ns, which is still shorter than the qubit's relaxation and dephasing times, e.g., T_1=7.3μs and T_2=500ns [28]. It is clear, however, that this is not enough to close the locality loophole. Therefore, our proposal should be experimentally realized with the current techniques.

In conclusion, we have proposed a simple and feasible method to test Bell's inequality with experimentally-demonstrated circuit QED system by joint spectral measurements. At first, an alternative method to generate an ideal Bell state is proposed with an iSWAP gate and several single-qubit gates [25]. Then we present a simple method to

confirm the generation of the Bell state with two single-qubit unitary operations and two projective measurements on two qubits. With the experimental joint spectral measurements, the confirmation can be easily achieved. Further, with two Hadamard-like operations constructed with single-qubit gates [25], two classical variables $\{\theta_1, \theta_2\}$ can be encoded into the generated Bell state. For the typical sets of classical local variables $\{\theta_j, \theta'_j, j = 1, 2\}$, the correlation functions in Bell's inequality can be directly read out by joint spectral measurements because each detected peak marks one of the logics states and its relative height corresponds to the superposed probability in the detected two-qubit states. In this way, the Bell's inequality could be efficiently tested. Compared with the previous schemes for testing Bell's inequality, the advantage of our proposal is that the desirable coincided measurements on the two qubits can be directly realized by joint spectral measurements and the nonlocal correlations between the distant qubits can be directly read out from the measured transmission spectra of the driven resonator. As a result, the present proposal seems much simpler and easier to be experimentally realized by joint spectral measurements. We believe that our method could be generalized to test Bell-like inequalities for multi-qubit states in a straightforward way.

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[1] A. Eistein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).
[2] J. S. Bell, Physics **1**, 195 (1964).
[3] J. F. Clauser, M. A. Horne, A. Simony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).
[4] See, eg., G. Wilhs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, Phys. Rev. Lett. **81**, 5039 (1998); A. Aspect, J. Dalibard, and G. Roger, *ibid* **49**, 1804 (1982); W. Title, J. Brendel, H. Zbinden, and N. Gisin, *ibid* **81**, 3563 (1998).
[5] M. A. Rowe, D. Kiepinski, V. Meyer, C. A. Sackett, W. M. Itano, C. Monroe, and D. J. Wineland, Nature (London) **409**, 791 (2001).
[6] D. L. Moehring, M. J. Madsen, B. B. Blinov, and C. Monroe, Phys. Rev. Lett. **93**, 090410 (2004).
[7] M. Ansmann, H. Wang, R. C. Bialczak, M. Hofheinz, E. Lucero, M. Neeley, A. D. O'Connell, D. Sank, M. Weides, J. Wenner, A. N. Cleland, and J. M. Martinis, Nature (London) **461**, 504 (2009).
[8] Y. Hasegawa, R. Loidl, G. Badurek, M. Baron, and H. Rauch, Nature (London) **425**, 45 (2003).
[9] A. Blais, R. S. Huang, A. Wallraff, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **69**, 062320 (2004).
[10] A. Wallraff, D.I. Schuster, A. Blais, L. Frunzlo, R. S. Huang, J. Majer, S. Kumar, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **431**, 162 (2004).
[11] D. I. Schuster, A. Wallraff, A. Blais, L. Frunzio, R.-S. Huang, J. Majer, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. Lett. **94**, 123602 (2005).
[12] P. J. Leek, J. M. Fink, A. Blais, R. Bianchetti, M. Goppl, J. M. Gambetta, D. I. Schuster, L. Frunzio, R. J. Schoelkopf, and A. Wallraff, Science **318**, 1889 (2007).
[13] A. Fragner, M. Goppl, J. M. Fink, M. Baur, R. Bianchetti, P. J. Leek, A. Blais, and A. Wallraff, Science **322**, 1357 (2008).
[14] A. A. Houck, D. I. Schuster, J. M. Gambetta, J. A. Schreier, B. R. Johnson, J. M. Chow, L. Frunzio, J. Majer, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **449**, 328 (2007).
[15] M. Hofheinz, E. M. Weig, M. Ansmann, R. C. Bialczak, E. Lucero, M. Neeley, A. D. O'Connell, H. Wang, J. M. Martinis, A. N. Cleland, Nature (London) **454**, 310 (2008).
[16] J. Majer, J. M. Chow, J. M. Gambetta, J. Koch, B. R. Johnson, J. A. Schreier, L. Frunzio, D. I. Schuster, A. A. Houck, A. Wallraff, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **449**, 443 (2007); M. A. Sillanpää, J. I. Park, and R. W. Simmonds, *ibid.* **449**, 438 (2007).
[17] L. DiCarlo, J. M. Chow, J. M. Gambetta, L. S. Bishop, B. R. Johnson, D. I. Schuster, J. Majer, A. Blais, L. Frunzio, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **460**, 240 (2009).
[18] D. I. Schuster, A. A. Houck, J. A. Schreier, A. Wallraff, J. M. Gambetta, A. Blais, L. Frunzio, J. Majer, B. Johnson, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Nature (London) **445**, 515 (2007).
[19] S. Filipp, P. Maurer, P. J. Leek, M. Baur, R. Bianchetti, J. M. Fink, M. Göppl, L. Steffen, J. M. Gambetta, A. Blais, and A. Wallraff, Phys. Rev. Lett. **102**, 200402 (2009).
[20] J. M. Chow, L. DiCarlo, J. M. Gambetta, A. Nunnenkamp. L. S. Bishop, L. Frunzio, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. A **81**, 062325 (2010).
[21] M. D. Reed, L. DiCarlo, B. R. Johnson, L. Sun, D. I. Schuster, L. Frunzio, and R. J. Schoelkopf, Phys. Rev. Lett. **105**, 173601 (2010).
[22] R. Bianchetti, S. Filipp, M. Göppl, P. J. Leek, L. Steffen, A. Blais, and A. Wallraff, Phys. Rev. A **80**, 043840 (2009).
[23] L. F. Wei, J. S. Huang, X. L. Feng, Z. D. Wang, and C. H. Oh, arXiv:1005.2470
[24] J. S. Huang, L. F. Wei, and C. H. Oh, arXiv:1009. 4252
[25] A. Blais, J. Gambetta, A. Wallraff, D. I. Schuster, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, Phys. Rev. A **75**, 032329 (2007).
[26] C. L. Hutchison, J. M. Gambetta, A. Blais, and F. K. Wilhelm, Can. J. Phys. **87**, 225 (2009).
[27] L. F. Wei, Y. X. Liu, and F. Nori, Phys. Rev. Lett. **96**, 246803 (2006).
[28] A. Wallraff, D. I. Schuster, A. Blais, L. Frunzio, J. Majer, M.H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Phys. Rev. Lett. **95**, 060501 (2005).
[29] L. F. Wei, Y. X. Liu, and F. Nori, Phys. Rev. B **72**, 104516 (2005); L. F. Wei, Y. X. Liu, M. J. Storcz, and F. Nori, Phys. Rev. A **73**, 052307 (2006).